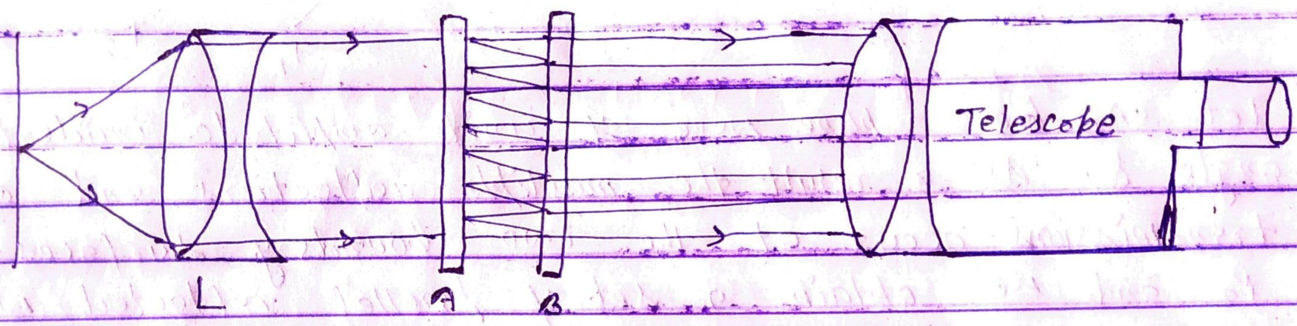
Fabry Perot Interferometer :→

In this interferometer the sharp Haidinger fringes are produced by means of multiple reflections from two partially silvered plates separated from each other.

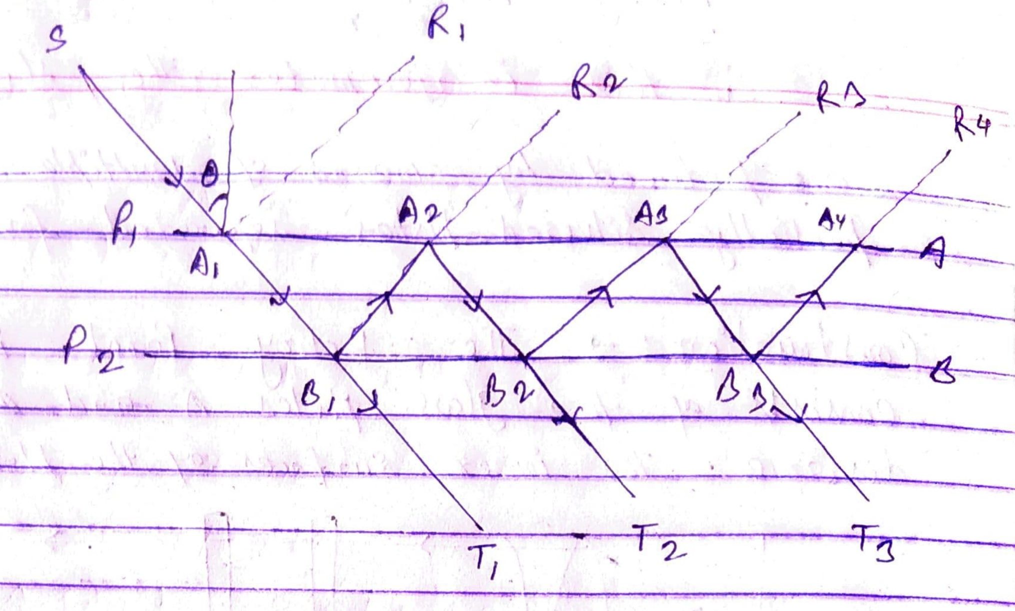
→ **Construction :→** The Fabry Perot interferometer essentially consists of two glass plates A and B separated by a distance. The inner surfaces of the plates are coated with



a thin film of aluminium which reflects about 75% of the incident light. The plate B facing the observer is fixed and is provided with screws with which the reflecting surface of B can be made parallel to that of A. The plate A is mounted on a carriage which can be moved in a direction ⊥r. to the reflecting surfaces by means of an accurate screw so that the thickness of air film between the coated surfaces of the plates A and B can be varied. Light from monochromatic extended source rendered parallel by collimating lens L, suffers multiple reflections in the air film between plates A and B. The transmitted light interferes and circular fringes of equal inclination are formed at the focal plane of the objective of the telescope.

→ **Theory of intensity distribution →**

In fig. P_1 and P_2 represent reflecting surfaces of the two plates A and B. Light from a monochromatic extended source is incident on the plates at all angles.



Let SA_1 be a plane wave of unit amplitude incident at an angle θ. As a result the multiple reflections and consequent transmissions occur at the two boundary surfaces P_1 and P_2 and we obtain a set of parallel reflected waves A_1R_1, A_2R_2, A_3R_3 ---- etc. and a set of parallel infinite transmitted beams B_1T_1, B_2T_2, B_3T_3 ---- etc. Let R and T be the fractions of intensity which are reflected and transmitted, i.e. the reflection and transmission coefficients. Obviously the fractions of amplitudes reflected and transmitted are $\sqrt{R}$ and $\sqrt{T}$. As the amplitude of the incident wave is unity, the amplitude of wave A_1B_1 is $\sqrt{T}$ and the amplitude of wave B_1T_1 is $(\sqrt{T} \times \sqrt{T}) = T$. The amplitude of wave B_1A_2 is $\sqrt{RT}$ and the amplitude of the wave A_2B_2 is $(\sqrt{R} \times \sqrt{RT}) = R\sqrt{T}$ and the amplitude of the wave B_2T_2 is RT. Similarly the amplitudes of waves B_3T_3, B_4T_4 etc. are R^2T, R^3T, ---- etc. Thus the amplitudes along B_1T_1, B_2T_2, B_3T_3, B_4T_4 etc. are T, RT, R^2T, ---- etc.

As all the transmitted waves B_1T_1, B_2T_2, B_3T_3 have been obtained from the same incident wave, they are capable of producing interference pattern.

Neglecting any phase change due to reflection from the silvered surfaces, there is a const. phase difference between any two consecutive transmitted beams due to path difference $2\mu t \cos\theta$.

$\therefore$ Phase diff. between two consecutive transmitted beams:

$$\delta = \frac{2\pi}{\lambda} \times 2\mu t \cos\theta$$

$$= \frac{2\pi}{\lambda} \cdot 2t \cos\theta \quad --- \quad (1) \quad (\because \text{for air film } \mu = 1)$$

Let the incident vibration be represented by

$$y' = 1 \cdot \sin\omega t$$

So that, the transmitted waves along B_1T_1, B_2T_2, B_3T_3, --- etc. at the point of interference will be represented by

$$y_1 = T \sin\omega t, \quad y_2 = TR \sin(\omega t - \delta)$$

$$y_3 = TR^2 \sin(\omega t - 2\delta), \quad y_4 = TR^3 \sin(\omega t - 3\delta) \ldots \text{ etc.}$$

If A is the amplitude of the resultant vibration and ϕ its phase difference, then

$$y = A \sin(\omega t - \phi)$$

By the principle of superposition, we have

$$y = y_1 + y_2 + y_3 + \; ----$$

or, $A \sin(\omega t - \phi) = T \sin\omega t + RT \sin(\omega t - \delta) + R^2 T \sin(\omega t - 2\delta)$ (2)

Expanding the sine terms and equating the coeffs. of $\sin\omega t$ and $\cos\omega t$, we get

$$A \cos\phi = T + TR\cos\delta + TR^2 \cos 2\delta + \ldots \quad (2)$$

$$A \sin\phi = TR \sin\delta + TR^2 \sin 2\delta + \ldots \quad (3)$$

As A is complex quantity, the resultant intensity is given by

$$I = A^2 = (A\cos\phi + iA\sin\phi)(A\cos\phi - iA\sin\phi) \quad \text{--- (4)}$$

where $i = \sqrt{-1}$

But from (2) & (3), we have

$$A\cos\phi + iA\sin\phi = T(1 + Re^{i\delta} + R^2 e^{2i\delta} + \cdots)$$

$$= \frac{T}{1 - Re^{i\delta}}$$

and $A\cos\phi - iA\sin\phi = T(1 + Re^{-i\delta} + R^2 e^{-2i\delta} - \cdots)$

$$= \frac{T}{1 - Re^{-i\delta}}$$

$\therefore$ Resultant intensity at the point of interference due to transmitted vibrations is

$$I = A^2 = \frac{T}{(1 - Re^{i\delta})} \times \frac{T}{(1 - Re^{-i\delta})}$$

$$= \frac{T^2}{1 + R^2 - 2R\cos\delta} = \frac{T^2}{(1-R)^2 + 2R - 2R\cos\delta}$$

$$= \frac{T^2}{(1-R)^2 + 2R(1 - \cos\delta)} = \frac{T^2}{(1-R)^2 + 4R\sin^2\frac{\delta}{2}}$$

$$= \frac{T^2}{(1-R)^2 + 4R\sin^2\frac{\delta}{2}}$$

$$= \frac{T^2}{(1-R)^2\left[1 + \frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2}\right]} \quad \text{--- (5)}$$

Thus the resultant intensity varies with R & δ.

→ Condition for maxima and minima : →

For maximum intensity $\sin^2\frac{\delta}{2} = 0$, i.e. $\frac{\delta}{2} = n\pi$

or, $\delta = 2n\pi = 0, 1, 2, 3,$ --- etc.

The maximum intensity will be $I_{max.} = \frac{T^2}{(1-R)^2}$ --- (6)

For minimum intensity,

$$\sin^2\frac{\delta}{2} = 1$$

$$\text{i.e. } \frac{\delta}{2} = (2n+1)\frac{\pi}{2}$$

$$\text{or, } \delta = (2n+1)\pi = (2n+1)\frac{\lambda}{2}$$

And the minimum intensity will be

$$I_{min.} = \frac{T^2}{(1-R)^2} \cdot \frac{1}{\left[1 + \frac{4R}{(1-R)^2}\right]}$$

$$= \frac{T^2}{(1+R)^2}$$

When there is no absorption for light in the plates,

$$T + R = 1$$

$$\text{or, } T = 1 - R \quad \text{--- (7)}$$

This gives, $I_{max.} = 1$

and $I_{min} = \frac{(1-R)^2}{(1+R)^2}$ ---- (8)

Using eqn (6), eqn (5) can be written as

$$I = I_{max} \cdot \frac{1}{1 + F \sin^2 \frac{\delta}{2}} \quad \text{--- (9)}$$

where $F = \frac{4R}{(1-R)^2}$ --- (10)

and is called the coefficient of fineness or coefficient of sharpness.

From eqn (9) $$\frac{I}{I_{max}} = \frac{1}{1 + F \sin^2 \frac{\delta}{2}} \quad \text{--- (11)}$$

The condition for maxima and minima in terms of path difference are

$$2t\cos\theta = n\lambda \quad \text{(maxima)}$$

$$2t\cos\theta = (2n-1)\frac{\lambda}{2} \quad \text{(minima)}$$

As 't' is constant, for a particular value of n and λ, θ must be constt. The locus of all points having same value of θ is a circle. Therefore the fringes are circular in shape.

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